

EXAMINATION 3

Directions: Do both problems, which have equal weight. This is a closed-book closed-note exam except for Griffiths, Pedrotti, a copy of anything posted on the course web site, and anything in your own handwriting (not a Xerox of someone else's writing). Calculators are not needed, but you may use one if you wish. Laptops and palmtops should be turned off. Use a bluebook. Do not use scratch paper – otherwise you risk losing part credit. Show all your work. Cross out rather than erase any work that you wish the grader to ignore. Justify what you do. Express your answer in terms of the quantities specified in the problem. Box or circle your answer.

Problem 1. (50 points)

Two point charges of charge $+2q$ each are located on the z axis at $(x, y, z) = (0, 0, a)$ and $(0, 0, -a)$. Four point charges of charge $-q$ each are located in the $z = 0$ plane on the x or y axes at $(x, y, z) = (a, 0, 0)$, $(0, a, 0)$, $(-a, 0, 0)$, and $(0, -a, 0)$.

For each of the two parts below, please focus on the *tangential (non-radial)* part \vec{E}_{tang} of the total electric field \vec{E} . Ignore octupole ($l = 3$) and higher multipoles. At observation point \vec{r} , for each part (a) and (b), please determine

- (i) (6 points) the dependence upon r of $|\vec{E}_{\text{tang}}|$ (in directions (θ, ϕ) where $|\vec{E}_{\text{tang}}|$ does not vanish);
- (ii) (6 points) within an overall sign, the direction ($\hat{\theta}$, $\hat{\phi}$, or some combination thereof) of \vec{E}_{tang} ; and
- (iii) (13 points) the polar angle(s) θ_{max} where, at a given radius, $|\vec{E}_{\text{tang}}|$ reaches its maximum value.

Please justify your determinations.

(a) (25 points)

For this part only, a is a constant. Take advantage of the approximation $a \ll r$ and work only to leading order in a/r .

(b) (25 points)

For this part only, $a = a_0(1 + \epsilon \cos \omega t)$, where $\epsilon \ll 1$, ω , and a_0 are constants. Taking advantage of the approximation $a \ll 2\pi c/\omega \ll r$, consider only the acceleration fields in the radiation (far) zone.

Problem 2. (50 points)

Consider a fully elliptically polarized beam “A” with the special property that the absolute magnitudes of its complex electric fields in the x and y directions are the *same*. If one ignores the overall phase of this beam, without loss of generality its Jones vector can be written

$$J_A = \sqrt{\frac{1}{2}} \begin{pmatrix} 1 \\ e^{i\psi} \end{pmatrix},$$

where ψ is a real constant.

A half-silvered mirror divides beam A into two components “A1” and “A2”. Beam A1 is not changed further. Beam A2 is transformed by an optical device into beam A3, such that the irradiance I_{A3} of beam A3 does not vanish. Thereafter, beams A3 and A1 are recombined with relative phase Δ .

Within an overall multiplicative constant, please design the device's Jones matrix M so that, for any phase Δ , beams A1 and A3 do not interfere at all:

$$I_{A1+A3} = I_{A1} + I_{A3}.$$

What physical optical element(s) would you use to build M ? Please specify the important physical parameter(s) of the element(s) [*e.g.* axis direction(s) or thickness(es)].